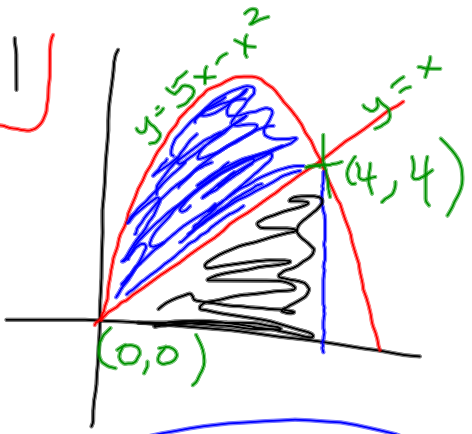
1




$\int_0^4 5x-x^2\,dx$

1) identify boundaries of region

$5x-x^2=x$

$4x-x^2=0$

$x(4-x)=0$

$x=0,4$

2) fn on top: $y=5x-x^2$

3) fn on bottom $y=x$

$$\text{Area}=\int_0^4 (5x-x^2)-(x)\,dx$$

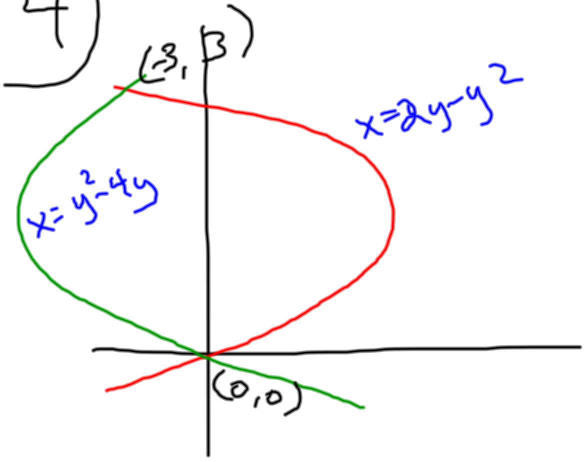
$$=\int_0^4 4x-x^2\,dx$$

fnInt$(4x-x^2, x, 0, 4)$
$=10.666\overline{6}$

$$\left(2x^2-\frac{x^3}{3}\right)\Big|_0^4=\left(32-\frac{64}{3}\right)-(0)$$

$$=\frac{32}{3}$$

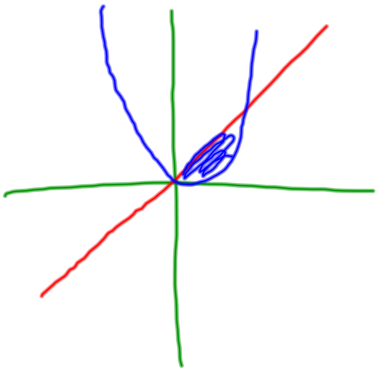
4)




$$\text{Area} = \int_0^3 f(x) - g(x)\,dy$$

$$= \int_0^3 (2y - y^2) - (y^2 - 4y)\,dy$$

7) $y = x$; $y = x^2$



1st $\int \sim dx$ or $\int \sim dy$?

2ND find boundaries

$x = x^2$

$x - x^2 = 0$

$x(1-x) = 0$

$\therefore x = 0$ ; $x = 1$

3rd set up def. int.

$$\text{Area} = \int_0^1 \text{top} - \text{bottom}\, dx$$

$$\text{Area} = \int_0^1 x - x^2\, dx$$

$$= \left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^1 = \left(\frac{1}{2} - \frac{1}{3}\right) - (0-0)$$

$$= \frac{1}{6}$$

1) $\int \sim\sim\sim \, dy$

2) find boundaries (y)

$y = x \Rightarrow x = y$

$y = x^2 \Rightarrow x = \sqrt{y}$

$y = \sqrt{y}$

$\Rightarrow y = 0, 1$

3) $\int_0^1 (\text{fn on right}) - (\text{fn on left}) \, dy$

$= \int_0^1 f(y) - g(y) \, dy$

$= \int_0^1 \sqrt{y} - y \, dy$

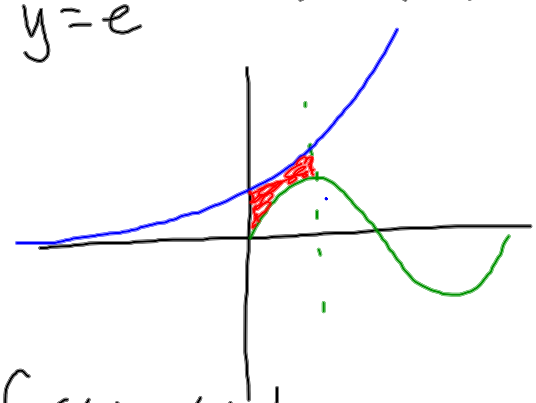
$= \left(\frac{2}{3} y^{3/2} - \frac{y^2}{2}\right)\Big|_0^1 = \left(\frac{2}{3} - \frac{1}{2}\right) - (0 - 0)$

$= \frac{1}{6}$

6)

$y = \sin x$ ; $x = 0$

$y = e^x$ ; $x = \frac{\pi}{2}$



$$\text{Area} = \int_0^{\pi/2} e^x - \sin x \, dx$$

$$\int f(y) - g(y) \, dy$$

$x = \frac{\pi}{2}$

$y = e^x \Rightarrow x = \ln y$

$y = \sin x \Rightarrow x = \sin^{-1}(y)$

Aim:

$y=f(x)$ $y=g(x)$

$$\text{Area} = \int_{a_x}^{b_x} f(x) - g(x)\, dx$$

OR

$x=f(y)$ $x=g(y)$

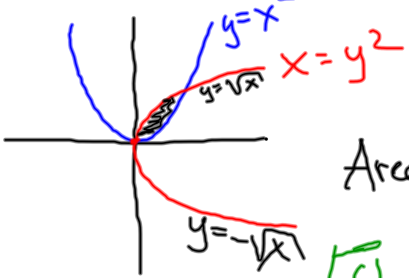
$$\text{Area} = \int_{c_y}^{d_y} f(y) - g(y)\, dy$$

11) $y^2 = x$ ; $x^2 = y$

$\int \quad dx$

$y = \sqrt{x}$ $\qquad$ $y = x^2$

$-\sqrt{x}$




Area $= \int_0^1 \sqrt{x} - x^2 \, dx$

$\sqrt{x} = x^2$?

$x = x^4$

$x - x^4 = 0$

$x(1-x^3) = 0$

$\int_0^1 x^{1/2} - x^2 \, dx$

$\left(\frac{2}{3}x^{3/2} - \frac{1}{3}x^3\right)\Big|_0^1$

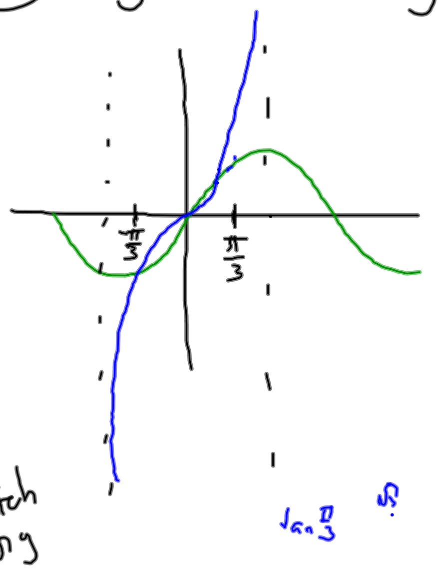
$= \left(\frac{2}{3} - \frac{1}{3}\right) - (0-0)$

$= \frac{1}{3}$

$x = f(y)$ $\qquad$ $\int \quad dy$ $\qquad$ $x^2 = y \Downarrow x = \sqrt{y}$ (not $-\sqrt{y}$)

$\int_0^1 \sqrt{y} - y^2 \, dy$

(5) $y = \tan x$; $y = 2\sin x$; $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$



sketch

x or y

draw a typical rect

why do we have to break this up?

$$2\sin(-x) = -2\sin(x)$$

$$\tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{\cos x} = -\tan x$$

$$\text{Area} = \int_{-\frac{\pi}{3}}^{0} (\tan x - 2\sin x)\,dx + \int_0^{\frac{\pi}{3}} (2\sin x - \tan x)\,dx$$

$$2\int_0^{\frac{\pi}{3}} 2\sin x - \tan x\,dx$$

$$= \int_0^{\pi/3} 2\sin x\,dx - \int_0^{\pi/3} \tan x\,dx$$

$$\left(-2\cos x\right)\Big|_0^{\pi/3} - \left(-\ln|\cos x|\right)\Big|_0^{\pi/3}$$

$\cos 0$

$$\int \tan x\,dx =$$

$$\int \frac{\sin x}{\cos x}\,dx$$

$u = \cos x$

$du = -\sin x\,dx$

$$-\int \frac{1}{u}\,du = -\ln|u| + C$$

$$= -\ln|\cos x| + C$$

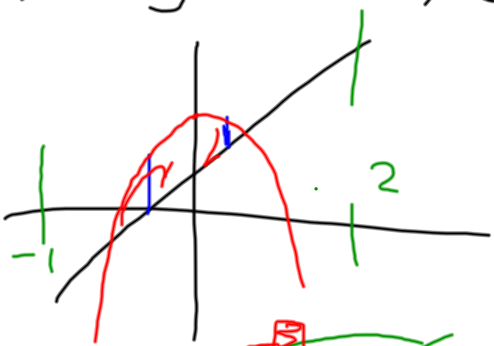
$$= \ln|\sec x| + C$$

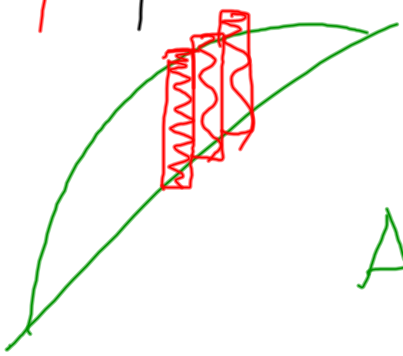
$$= -2\left(\tfrac{1}{2}\right) - (-2) - \left(-\ln\left|\tfrac{1}{2}\right| + (\ln 1)\right)$$

$$= -1 + 2 + \ln\tfrac{1}{2} - 0$$

$$= 1 + \ln\tfrac{1}{2}$$

5) $y = x+1$, $y = 9 - x^2$, $x = -1$, $x = 2$



$$9 - x^2 = x + 1$$

$$x^2 + x - 8 = 0$$

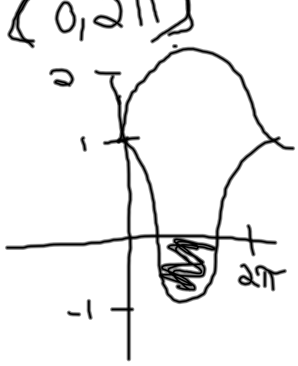
$$x = \frac{-1 \pm \sqrt{1+32}}{2}$$

$$= \frac{\pm\sqrt{33} - 1}{2}$$

$$\text{Area} = \int_{-1}^{2} (9 - x^2) - (x+1)\,dx$$

14)

$y = \cos x$ ; $2 - \cos x$ $[0, 2\pi]$




$$\int_0^{2\pi} (2 - \cos x) - \cos x \, dx$$

$$\int_0^{2\pi} 2 - 2\cos(x) \, dx$$

$$2x - 2\sin(x) + C \Big|_0^{2\pi}$$

$$[2(2\pi) - 2(\sin(2\pi) + C] - 2(0) - 2\sin(0) + C)$$

$$[4\pi - 0] - [0]$$

Area $= 4\pi$

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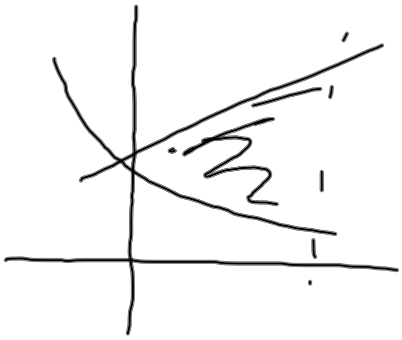
2) Find the area between $y = \sqrt{x+2}$ and $y = \frac{1}{x+1}$ & between $x=0$ & $x=2$



$$\text{Area} = \int_0^2 \sqrt{x+2} - \frac{1}{x+1}\,dx$$

$$= \int_0^2 \sqrt{x+2}\,dx - \int_0^2 \frac{1}{x+1}\,dx$$

$u = x+2$

$\frac{du}{dx} = 1$

$du = dx$

when $x=0$, $u=2$

when $x=2$, $u=4$

$$\int_2^4 u^{1/2}\,du$$

$$= \frac{2}{3}u^{3/2}\Big|_2^4$$

$u = x+1$

$\frac{du}{dx} = 1$

$du = dx$

when $x=0$, $u=1$

when $x=2$, $u=3$

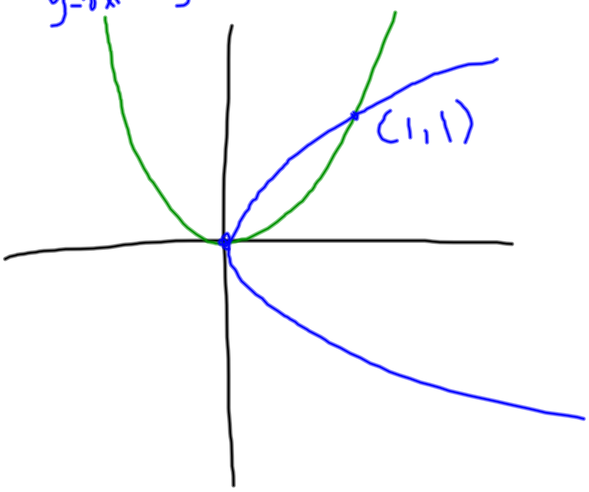
$$\int_1^3 \frac{1}{u}\,du = \ln|u|\Big|_1^3$$

$$= \frac{2}{3}(\sqrt{4})^3 - \frac{2}{3}(\sqrt{2})^3 - (\ln 3 - \ln 1)$$

$$\frac{16}{3} - \frac{4}{3}\sqrt{2} - \ln 3$$

11) $y^2 = x$ ; $x^2 = y$

$y = \sqrt{x}$ and $y = -\sqrt{x}$



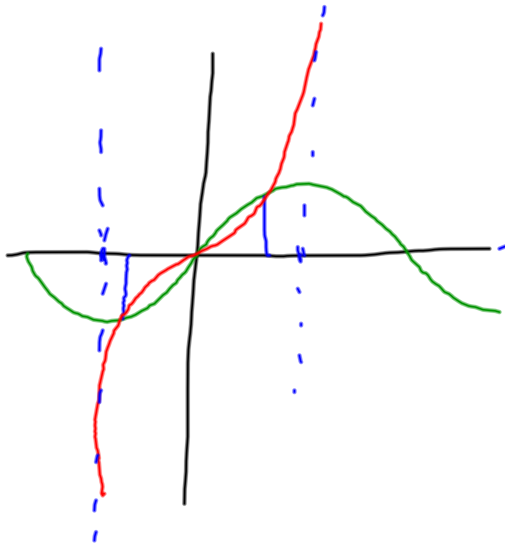

$$\text{Area} = \int_0^1 \underbrace{\sqrt{x}}_{x^{1/2}} - x^2 \, dx$$

$$= \left( \frac{2}{3} x^{3/2} - \frac{x^3}{3} \right) \Big|_0^1$$

$$= \left( \frac{2}{3} - \frac{1}{3} \right) - (0 - 0)$$

$$= \frac{1}{3}$$

15) $y = \tan x$ ; $y = 2\sin x$ ; $-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$



$2\sin(-x) = -2\sin x$

$\tan(-x) = -\tan x$

$$2\int_0^{\pi/3} 2\sin x - \tan x \, dx$$